LESSON 5.1a

nth Roots and Rational Exponents

Today you will:

- Find n^{th} roots of numbers.
- Evaluate expressions with rational exponents.
- Practice using English to describe math processes and equations

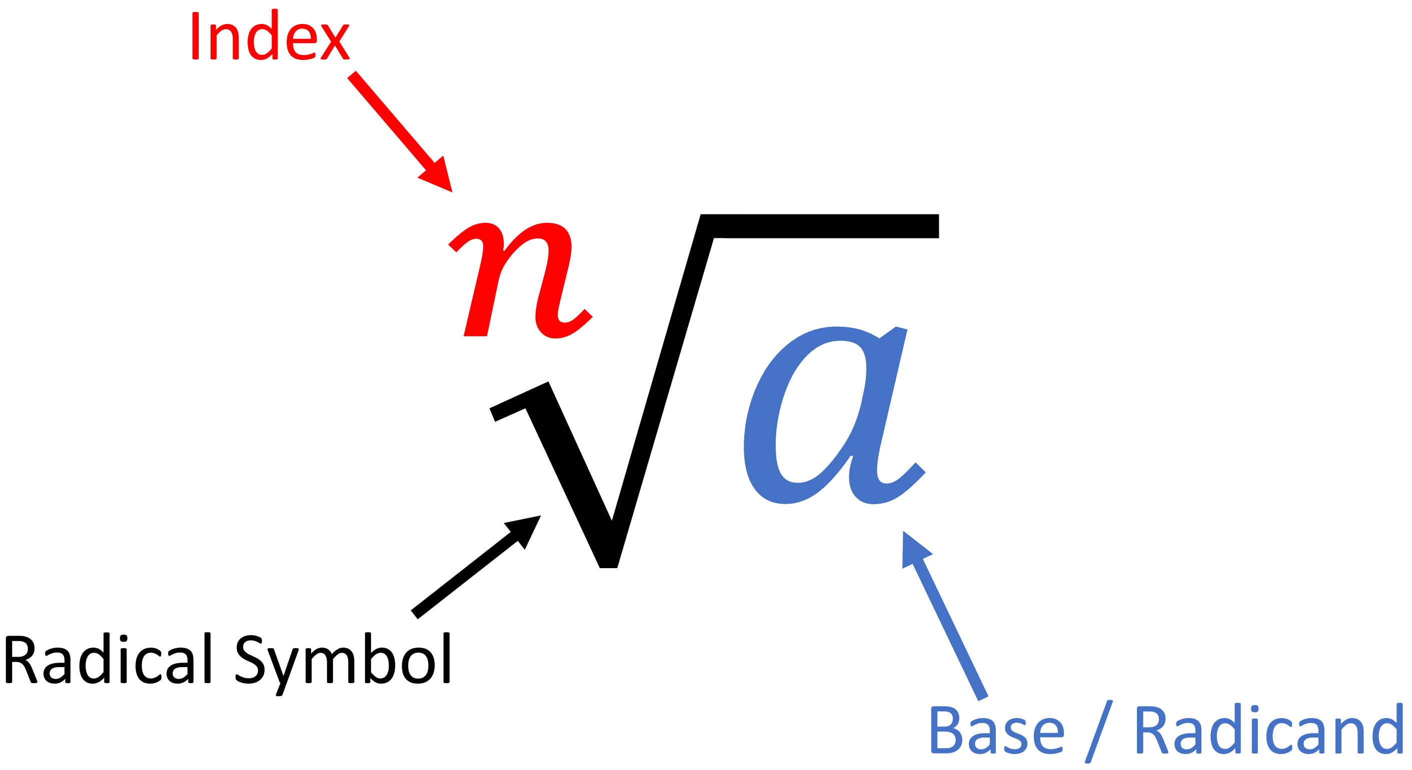
Core Vocabulary:

- radical form
- nth root of a, p. 238
- index of a radical, p. 238

Previous:

- square root
- cube root
- exponent

Radical Form



$$\sqrt{4} = \sqrt[?]{4} = \sqrt[2]{4} = square\ root\ of\ 4 = 4^{?} = 4^{\frac{1}{2}}$$

Radical Form

Rational Exponent Form

Radical Form	How we say it	Rational Exponent Form
$\sqrt[2]{a}$	Square root of a	$a^{\frac{1}{2}}$
$\sqrt[3]{a}$	Cube root of a	$a^{\frac{1}{3}}$
$\sqrt[4]{a}$	4^{th} root of a	$a^{\frac{1}{4}}$
	…	
$\sqrt[n]{a}$	n^{th} root of a	$a^{\frac{1}{n}}$

What does $\sqrt[2]{25}$ mean?

- Find the square root of 25.
- Find the number that times itself **2** times equals **25**.

What does $\sqrt[5]{32}$ mean?

- Find the 5th root of 32.
- Find the number that times itself **5** times equals **32**.

What does $\sqrt[n]{a}$ mean?

- Find the ***n***th root of ***a***.
- Find the number that times itself ***n*** times equals ***a***.

It is ****REALLY**** helpful to know the 1st few powers of up to 5…

Power	Powers of 2	Powers of 3	Powers of 4	Powers of 5
1	2	3	4	5
2	4	9	16	25
3	8	27	64	125
4	16	81	256	
5	32	243		
6	64			

UNDERSTANDING MATHEMATICAL TERMS

When n is even and $a > 0$, there are two real roots. The positive root is called the *principal root*.

Core Concept

Real *n*th Roots of *a*

Let n be an integer $(n > 1)$ and let a be a real number.

	***n* is an even integer.**		***n* is an odd integer.**
$a < 0$	No real nth roots	$a < 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$
$a = 0$	One real nth root: $\sqrt[n]{0} = 0$	$a = 0$	One real nth root: $\sqrt[n]{0} = 0$
$a > 0$	Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	$a > 0$	One real nth root: $\sqrt[n]{a} = a^{1/n}$

Find the indicated real *n*th root(s) of *a*.

a. $n = 3$, $a = -216$

b. $n = 4$, $a = 81$

Remember:

An *n*th root of *a* is written as $\sqrt[n]{a}$.

...and $\sqrt[n]{a} = a^{1/n}$

Answer in radical form

Answer in rational exponent form

SOLUTION

a. Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root.

Because $(-6)^3 = -216$, you can write $\sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.

b. Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots.

Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\sqrt[4]{81} = \pm 3$ or $81^{1/4} = \pm 3$.

Some basic properties of Radicals

These work:

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \quad because \sqrt[n]{a^m} = (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}}$$

$$\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

These ***DO NOT*** work:

$$\sqrt[n]{a} + \sqrt[n]{b} \neq \sqrt[n]{a+b}$$

$$\sqrt[n]{a} - \sqrt[n]{b} \neq \sqrt[n]{a-b}$$

EXAMPLE 2 Evaluating Expressions with Rational Exponents

Evaluate each expression.

a. $16^{3/2}$ **b.** $32^{-3/5}$

SOLUTION

Rational Exponent Form	**Radical Form**
a. $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$	$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$
b. $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(32^{1/5})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$	$32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(\sqrt[5]{32})^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

Using a calc to evaluate radicals

1. Convert the radical to rational exponent form.
2. Then enter the base in the calculator.
3. Hit the “^” key.
4. Finally enter the fractional exponent ***inside parentheses***.

Evaluate each expression using a calculator. Round your answer to two decimal places.

COMMON ERROR

Be sure to use parentheses to enclose a rational exponent: 9^(1/5) ≈ 1.55. Without them, the calculator evaluates a power and then divides: 9^1/5 = 1.8.

a. $9^{1/5}$ **b.** $12^{3/8}$ **c.** $\left(\sqrt[4]{7}\right)^3$

SOLUTION

a. $9^{1/5} \approx 1.55$

b. $12^{3/8} \approx 2.54$

c. Before evaluating $\left(\sqrt[4]{7}\right)^3$, rewrite the expression in rational exponent form.

$$\left(\sqrt[4]{7}\right)^3 = 7^{3/4} \approx 4.30$$

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9^(1/5)
            1.551845574
12^(3/8)
            2.539176951
7^(3/4)
            4.303517071
```

Homework

Pg 241, #1-34